## **COURSE OUTLINE**

#### **GENERAL**

| SCHOOL   | Sciences and Engineering      |                             |         |  |
|--|-------------------------------|-----------------------------|---------|--|
| ACADEMIC UNIT  | Computer Science              |                             |         |  |
| LEVEL OF STUDIES   | 1 <sup>st</sup> Cycle         |                             |         |  |
| COURSE CODE  | MATH-335 <b>SEMESTER</b> Fall |                             |         |  |
| COURSE TITLE   | Optimization Techniques       |                             |         |  |
| if credits are awarded for separate components of the course, e.g. lectures, laboratory exercises, etc. If the credits are awarded for the whole of the course, give the weekly teaching hours and the total credits |                               | WEEKLY<br>TEACHING<br>HOURS | CREDITS |  |
|  |                               | 2.5                         | 6       |  |
|  |                               |                             |         |  |
|  |                               |                             |         |  |
| Add rows if necessary. The organisation of teaching and the teaching methods used are described in detail at (d).  |                               |                             |         |  |
| COURSE TYPE general background, special background, specialised general knowledge, skills development  | Special background            |                             |         |  |
| PREREQUISITE COURSES:  | MATH-196, MATH-280            |                             |         |  |
| LANGUAGE OF INSTRUCTION and EXAMINATIONS:  | English                       |                             |         |  |
| IS THE COURSE OFFERED TO ERASMUS STUDENTS  |                               |                             |         |  |
| COURSE WEBSITE (URL)   |                               |                             |         |  |

# **LEARNING OUTCOMES**

#### **Learning outcomes**

The course learning outcomes, specific knowledge, skills and competences of an appropriate level, which the students will acquire with the successful completion of the course are described.

## Consult Appendix A

- Description of the level of learning outcomes for each qualifications cycle, according to the Qualifications Framework of the European Higher Education Area
- Descriptors for Levels 6, 7 & 8 of the European Qualifications Framework for Lifelong Learning and Appendix B
- Guidelines for writing Learning Outcomes

After completion of the course students are expected to be able to:

- Visualize concepts from Linear Algebra and Optimization with the aid of Geometry.
- Compute Matrix Factorizations and apply them to dimensionality reduction and Image Compression problems.
- Formulate and solve Least-Squares problems in the context of Linear Regression.
- Compute partial derivatives and gradients of multivariable functions.
- Find the local and global extrema of multivariable functions using the Hessian and relevant derivative criteria.
- Apply Gradient Descent to univariate and multivariate optimization problems.

 Employ Linear Algebra and Mathematical Optimization techniques to Data Science and Machine Learning

# **General Competences**

Taking into consideration the general competences that the degree-holder must acquire (as these appear in the Diploma Supplement and appear below), at which of the following does the course aim?

Search for, analysis and synthesis of data and information, with the use of the necessary technology

Adapting to new situations Decision-making

Working independently

Team work
Working in an international environment

Working in an interdisciplinary environment

Production of new research ideas

Project planning and management
Respect for difference and multiculturalism
Respect for the natural environment

Showing social, professional and ethical responsibility and sensitivity to

gender issues

Criticism and self-criticism

Production of free, creative and inductive thinking

Others...

Analysis and synthesis of data with the use of the necessary technology, adapting to new situations, decision-making, working independently, working in an interdisciplinary environment, analytical, critical and quantitative thinking, synthesis of ideas.

#### **SYLLABUS**

- 1.Introduction (Motivation; Some examples of Optimization problems).
- 2. Review of Matrices and Linear Systems:
  - a. Linear Systems (Geometry and matrix representation)
  - b. Matrix Fundamentals (Basic operations, Matrices as linear maps, Determinants, Matrices with special structure and properties.
  - c. Solution of linear systems using Gaussian elimination.

#### 3. Vector Spaces:

- a. Subspaces and affine sets.
- b. Review: linear independence, basis and dimension, rank of a matrix, the rank and nullity theorem.
- c. Inner products; Vector and Matrix Norms and their geometric interpretation; Orthogonality and Projections; The Gram-Schmidt procedure.

#### 4. Matrix Factorizations:

- a. Eigenvalues and Eigenvectors
- b. Geometric and Algebraic multiplicity, eigenspaces.
- c. The LU and Cholesky decompositions.
- d. Eigendecomposition and Diagonalization
- e. Orthogonal diagonalization of a symmetric matrix. Spectral decomposition and dimensionality reduction.
- f. The SVD. Image compression and other applications.

## 5. Continuous Optimization Problems and Gradient Descent:

- a. Univariate Optimization (Taylor's Theorem, Univariate Gradient Descent: Convergence and Divergence)
- b. Multivariable functions and partial derivatives. The Gradient vector.
- c. Multivariate Optimization. (Taylor's Theorem, Local and Global Extrema, the Hessian)

- d. Convexity. (Convex sets and Convex functions, the First and Second Derivative Conditions)
- e. Gradient descent. (Checking with the aid of Finite Differences, Decaying Learning Rates, Line Search, Stochastic Gradient Descent, Typical Objective Functions in Machine Learning)
- f. Least squares as an optimization problem. Regression.

## **TEACHING and LEARNING METHODS - EVALUATION**

| <b>DELIVERY</b> Face-to-face, Distance learning, etc.  | Face-to-face   |                   |  |
|--|--|-------------------|--|
| USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY Use of ICT in teaching, laboratory education, communication with students   | Use of ICT in teaching / Χρήση ΤΠΕ<br>Communication with students / Επικοινωνία με Φοιτητές  |                   |  |
| TEACHING METHODS   |  |                   |  |
| The manner and methods of teaching are described in detail.  | Activity   | Semester workload |  |
| Lectures, seminars, laboratory practice,   | Lectures   | 35                |  |
| fieldwork, study and analysis of bibliography,   | Practice problems  | 46                |  |
| tutorials, placements, clinical practice, art workshop, interactive teaching, educational  | Written and  | 21                |  |
| visits, project, essay writing, artistic creativity, etc.  | programming  |                   |  |
|  | assignments  | _                 |  |
| The student's study hours for each learning activity are given as well as the hours of non-  | Study of the textbook,   | 48                |  |
| directed study according to the principles of the  | lecture notes and online   |                   |  |
| ECTS   | material Course total  | 150               |  |
| CTUDENT DEDECTARANCE   | Course total   | 150               |  |
| STUDENT PERFORMANCE EVALUATION   |  |                   |  |
| Description of the evaluation procedure  Language of evaluation, methods of evaluation, summative or conclusive, multiple choice questionnaires, short-answer questions, openended questions, problem solving, written work, essay/report, oral examination, public presentation, laboratory work, clinical examination of patient, art interpretation, other  Specifically-defined evaluation criteria are given, and if and where they are accessible to students. | <ul> <li>- Final Examination</li> <li>- Midterm Examination</li> <li>- Assignments (written and programming)</li> <li>- Participation</li> </ul> |                   |  |

## **ATTACHED BIBLIOGRAPHY**

| Required Textbooks / Read | dings:    |           |      |      |
|---------------------------|-----------|-----------|------|------|
| Title                     | Author(s) | Publisher | Year | ISBN |

| Mathematics for Machine Learning                                 | Marc Peter<br>Deisenroth, A. Aldo         | Free Online<br>Version /         | 2021 | https://mml-<br>book.com.                  |
|--|---|----------------------------------|------|--|
|  | Faisal, Cheng Soon Ong                    | Cambridge<br>University<br>Press | 2020 | 9781108455145<br>(hardcopy, in<br>library) |
| Linear Algebra and Optimization for Machine Learning: A Textbook | Charu C. Aggarwal                         | Springer                         | 2020 | 9783030403447<br>(e-book)                  |
| Elementary Linear<br>Algebra: Applications<br>Version            | Howard Anton, Chris<br>Rorres, Anton Kaul | Wiley 12 <sup>th</sup><br>Ed.    | 2019 | 9781119670766<br>(e-book)                  |

# **Recommended Textbooks / Readings:**

| Title                 | Author(s)        | Publisher                   | Year | ISBN                       |
|-----------------------|------------------|-----------------------------|------|----------------------------|
| Introduction to       | Stephen Boyd,    | Cambridge                   | 2018 | https://web.stanford.edu   |
| applied linear        | Lieven           | University                  |      | /<br>~hovd/vmls/           |
| algebra:Vectors,      | Vandenberghe     | Press                       |      | <u>~boyd/vmls/</u>         |
| Matrices and          |                  |                             |      | (e-book, available online) |
| Least Squares         |                  |                             |      |                            |
| Elementary Linear     | B. Kolman and D. | Pearson 9 <sup>th</sup> Ed. | 2017 | 9780134718538              |
| Algebra               | Hill             |                             |      |                            |
|                       |                  |                             |      |                            |
| Optimization for Data | Stephen Wright   | Cambridge                   | 2022 | 9781009004282              |
| Analysis              | and Benjamin     | University                  |      | (e-book)                   |
|                       | Recht            | Press                       |      |                            |