UNIVERSITY of NICOSIA

Course Syllabus

Course Code	Course Title	ECTS Credits
MATH-335	Optimization Techniques	6
Prerequisites	**Department**	**Semester**
MATH-191, MATH-280	Computer Science	Fall
Type of Course	**Field**	**Language of Instruction**
Required	Mathematics	English
Level of Course	**Lecturer(s)**	**Year of Study**
1st Cycle	Nectarios Papanicolaou	3rd
Mode of Delivery	**Work Placement**	**Corequisites**
Face-to-face	N/A	None

Course Objectives:

The main objectives of the course are to:

- Introduce students to fundamental Optimization concepts.
- Build on students' existing Linear Algebra knowledge in order to provide the necessary tools to analyze models.
- Develop geometric intuition in the context of Linear Algebra and Optimization.
- Introduce students to Matrix Factorizations and their applications.
- Familiarize students with Least Squares and Linear Regression.
- Enhance multivariate thinking through the fundamentals of Linear and Nonlinear Convex Optimization.
- Introduce Gradient Descent and cover fundamental concepts and definitions.
- Discuss how the Linear Algebra and Optimization techniques covered are used in a Machine Learning or Data Analysis context.

Learning Outcomes:

After completion of the course students are expected to be able to:

1. Visualize concepts from Linear Algebra and Optimization with the aid of Geometry.
2. Compute Matrix Factorizations and apply them to dimensionality reduction and Image Compression problems.
3. Formulate and solve Least-Squares problems in the context of Linear Regression.
4. Compute partial derivatives and gradients of multivariable functions.

5. Find the local and global extrema of multivariable functions using the Hessian and relevant derivative criteria.
6. Apply Gradient Descent to univariate and multivariate optimization problems.
7. Employ Linear Algebra and Mathematical Optimization techniques to Data Science and Machine Learning

Course Content:

1. Introduction (Motivation; Some examples of Optimization problems).

2. Review of Matrices and Linear Systems:
   a. Linear Systems (Geometry and matrix representation)
   b. Matrix Fundamentals (Basic operations, Matrices as linear maps, Determinants, Matrices with special structure and properties.
   c. Solution of linear systems using Gaussian elimination.

3. Vector Spaces:
   a. Subspaces and affine sets.
   b. Review: linear independence, basis and dimension, rank of a matrix, the rank and nullity theorem.
   c. Inner products; Vector and Matrix Norms and their geometric interpretation; Orthogonality and Projections; The Gram-Schmidt procedure.

4. Matrix Factorizations:
   a. Eigenvalues and Eigenvectors
   b. Geometric and Algebraic multiplicity, eigenspaces.
   c. The LU and Cholesky decompositions.
   d. Eigendecomposition and Diagonalization
   e. Orthogonal diagonalization of a symmetric matrix. Spectral decomposition and dimensionality reduction.
   f. The SVD. Image compression and other applications.

5. Continuous Optimization Problems and Gradient Descent:
   a. Univariate Optimization (Taylor's Theorem, Univariate Gradient Descent: Convergence and Divergence)
   b. Multivariable functions and partial derivatives. The Gradient vector.
   c. Multivariate Optimization. (Taylor's Theorem, Local and Global Extrema, the Hessian)
   d. Convexity. (Convex sets and Convex functions, the First and Second Derivative Conditions)
   e. Gradient descent. (Checking with the aid of Finite Differences, Decaying Learning Rates, Line Search, Stochastic Gradient Descent, Typical Objective Functions in Machine Learning)
   f. Least squares as an optimization problem. Regression.

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Learning Activities and Teaching Methods:

Lectures, Handouts, Online Material, Assignments, In-class Exercises.

Assessment Methods:

Final Examination, Midterm Examination, Assignments (written and programming) and Participation.

Required Textbooks / Readings:

Title	Author(s)	Publisher	Year	ISBN
Mathematics for Machine Learning	Marc Peter Deisenroth, A. Aldo Faisal, Cheng Soon Ong	Free Online Version / Cambridge University Press	2021 2020	https://mml-book.com. 9781108455145 (hardcopy, in library)
Linear Algebra and Optimization for Machine Learning: A Textbook	Charu C. Aggarwal	Springer	2020	9783030403447 (e-book)
Elementary Linear Algebra: Applications Version	Howard Anton, Chris Rorres, Anton Kaul	Wiley 12th Ed.	2019	9781119670766 (e-book)

Recommended Textbooks / Readings:

Title	Author(s)	Publisher	Year	ISBN
Introduction to applied linear algebra:Vectors, Matrices and Least Squares	Stephen Boyd, Lieven Vandenberghe	Cambridge University Press	2018	https://web.stanford.edu/~boyd/vmls/ (e-book, available online)
Elementary Linear Algebra	B. Kolman and D. Hill	Pearson 9th Ed.	2017	9780134718538

Optimization for Data Analysis	Stephen Wright and Benjamin Recht	Cambridge University Press	2022	9781009004282 (e-book)