Course Code MATH-441	**Course Title** Numerical Differential Equations	**ECTS Credits** 8
Department Mathematics	**Semester** Fall or Spring	**Prerequisites** MATH-342, MATH-330, MATH-430
Type of Course Elective	**Field** Mathematics	**Language of Instruction** English
Level of Course 1st Cycle	**Year of Study** 4th	**Lecturer(s)** Dr Nectarios Papanicolaou Dr. Marios Christou
Mode of Delivery Face-to-face	**Work Placement** N/A	**Co-requisites** None

Objectives of the Course:

The main objectives of the course are to:

- Cover one-step methods for first-order initial value problems in detail.
- Familiarize students with the notion of truncation error.
- Present multistep methods for first-order ODEs and discuss their stability.
- Introduce students to the numerical solution of two-point boundary value problems.
- Cover Finite Difference methods for parabolic PDEs in depth. Analyze their stability using Fourier analysis.
- Develop Finite Difference methods for first order hyperbolic PDEs.
- Introduce students to numerical methods for the Laplace and Poisson equations

Learning Outcomes:

After completion of the course students are expected to be able to:

- Derive finite difference schemes for initial value problems.
- Compute the truncation error and convergence rate of these schemes.
- Employ the Dahlquist equaivalence theorem to assess the stability of multistep schemes.
- Implement various finite difference methods for partial differential equations.
- Assess the stability of finite difference schemes for evolution equations using Fourier analysis (von Neumann condition).
- Implement the derived algorithms using high-level programming languages. Critically assess the results.

Course Contents:

1. First-order initial value problems
    - Review of theory
    - The Explicit and Implicit Euler Methods.
    - The trapezoidal and theta methods
    - Stability and Truncation error
2. Higher-Order Methods
    - Runge-Kutta Methods
    - Multistep methods
        - Adams-Bashforth
        - Adams-Moulton
    - Error and Stability-Dahlquist's Theorems
3. The Two-Point Boundary Value Problem
    - Linear BVPs
    - Nonlinear BVPs
    - Shooting Method
4. Systems of first-order ODE's
5. Finite difference methods for Parabolic Equations
    - Parabolic equations in 1D
        - Explicit Schemes and convergence
        - The theta method and the Crank-Nicolson scheme
    - Parabolic equations in 2D and 3D
        - An explicit method
        - ADI methods
    - Fourier stability analysis
6. Finite difference methods for Hyperbolic equations
    - Hyperbolic equations in 1D
        - Characteristics
        - The CFL condition
        - The upwind scheme
        - The Lax-Wendroff scheme
        - The leap-frog scheme
    - Fourier stability analysis
7. Finite difference methods for Elliptic PDEs
    - The Laplace equation
    - The Poisson Equation

Learning Activities and Teaching Methods:

Lectures, Homework and Programming Assignments

Assessment Methods:

Homework, Mid-Term Exam, Programming Assignments, Final Exam

Required Textbooks/Reading:

Authors	Title	Publisher	Year	ISBN
L. Edsberg	Introduction to Computation and Modeling for Differential Equations	Wiley	2008	0470270853

Recommended Textbooks/Reading:

Authors	Title	Publisher	Year	ISBN
K. E. Atkinson, W, Han, D.E. Stewart	Numerical Solution of Ordinary Differential Equations	Wiley	2009	047004294X
J. C. Strikwerda	Finite Difference Schemes and Partial Differential Equations (2nd edition)	SIAM	2007	089871639X